



TITLE:

Pointwise and Sequential Continuity in Constructive Analysis (Algebra, Logic and Geometry in Informatics)

AUTHOR(S):

Ishihara, Hajime

CITATION:

Ishihara, Hajime. Pointwise and Sequential Continuity in Constructive Analysis (Algebra, Logic and Geometry in Informatics). 数理解析研究所講究録 2003, 1318: 1-2

ISSUE DATE:

2003-05

URL:

<http://hdl.handle.net/2433/43023>

RIGHT:

Pointwise and Sequential Continuity in Constructive Analysis

Hajime Ishihara (石原 哉)
JAIST (北陸先端科学技術大学院大学)

We discuss various continuity properties, especially pointwise and sequential continuity, in Bishop's constructive mathematics; see [1, 2, 11] for Bishop's constructive mathematics and [3, 4, 5, 9] for various continuity properties. We say that a mapping f between metric spaces X and Y is *sequentially continuous* if $x_n \rightarrow x$ implies that $f(x_n) \rightarrow f(x)$; *pointwise continuous* if for each $x \in X$ and $\epsilon > 0$ there exists $\delta > 0$ such that $d(x, y) < \delta$ implies $d(f(x), f(y)) < \epsilon$ for all $y \in X$. We first show the following theorem.

Theorem 1 *The following are equivalent.*

1. *Every sequentially continuous mapping of a separable metric space into a metric space is pointwise continuous.*
2. *Every sequentially continuous mapping of a complete separable metric space into a metric space is pointwise continuous.*
3. *BD-N. Every countable pseudo-bounded subset of \mathbb{N} is bounded.*

Here a subset A of \mathbb{N} is said to be *pseudo-bounded* if for each sequence $\{a_n\}$ in A , $a_n < n$ for all sufficiently large n . Note that although BD-N holds in classical mathematics, intuitionistic mathematics and constructive recursive mathematics of Markov's school, a natural recursivisation of BD-N is independent of Heyting arithmetic [3, 5, 8, 10].

We also show that very important theorems in functional analysis – Banach's inverse mapping theorem, the open mapping theorem, the closed graph theorem, the Banach-Steinhaus theorem and the Hellinger-Toeplitz theorem – can be proved in Bishop's constructive mathematics for *sequentially continuous* linear mappings [6, 7]. However it has emerged that the theorems for *pointwise continuous* linear mappings are equivalent to BD-N

参考文献

- [1] Errett Bishop, *Foundations of Constructive Analysis*, McGraw-Hill, New York, 1967.
- [2] Errett Bishop and Douglas Bridges, *Constructive Analysis*, Springer-Verlag, Heidelberg, 1985.
- [3] Douglas Bridges, Hajime Ishihara, Peter Schuster and Luminița Viță, *Strong continuity implies uniform sequential continuity*, preprint, 2001.
- [4] Hajime Ishihara, *Continuity and nondiscontinuity in constructive mathematics*, J. Symbolic Logic **56** (1991), 1349–1354.
- [5] Hajime Ishihara, *Continuity properties in constructive mathematics*, J. Symbolic Logic **57** (1992), 557–565.
- [6] Hajime Ishihara, *A constructive version of Banach's inverse mapping theorem*, New Zealand J. Math. **23** (1994), 35–43.
- [7] Hajime Ishihara, *Sequential continuity of linear mappings in constructive mathematics*, J. UCS **3** (1997), 1250–1254.
- [8] Hajime Ishihara, *Sequential continuity in constructive mathematics*, In: C.S. Calude, M.J. Dinneen and S. Sburlan eds., *Combinatorics, Computability and Logic*, Springer-Verlag, London, 2001, 5–12.
- [9] Hajime Ishihara and Peter Schuster, *Some constructive uniform continuity theorem*, Q. J. Math. **53** (2002), 185–193.
- [10] Hajime Ishihara and Satoru Yoshida, *A constructive look at the completeness of $\mathcal{D}(\mathbf{R})$* , J. Symbolic Logic **67** (2002), 1511–1519.
- [11] A. S. Troelstra and D. van Dalen, *Constructivism in Mathematics*, Vol. 1 and 2, North-Holland, Amsterdam, 1988.